American Higher Education and Income Inequality

Catharine B. Hill

Vassar College

124 Raymond Avenue
Poughkeepsie, N.Y. 12604

845-437-7200

chill@vassar.edu
American Higher Education and Income Inequality

Catharine Hill

President and Professor of Economics

Vassar College

Abstract

This paper demonstrates that increasing income inequality can contribute to the trends we see in American higher education and to the financial challenges that many colleges and universities are currently facing. The discussion is most appropriate for the selective non-profit higher education institutions, but also applies to selective publics with minor changes. Given their selective admissions and commitment to the socioeconomic diversity of their students, it demonstrates how increasing income inequality leads to higher tuition, higher costs, and higher financial aid than would otherwise be the case. A simple numerical example is also presented that estimates how much lower tuition, spending (a proxy for costs), and financial aid would have been, if household incomes in the United States had grown by the same aggregate amount between 1971 and 2009, but with no increase in income inequality. The policy implications are also discussed. The government is in the best position to address rising income inequality directly. At the same time, policy makers could also change the incentives facing higher education by more carefully targeting subsidies and tax expenditures, to help insure that the education system contributes to, rather than worsens, future income inequality. This paper will be of interest to policy makers interested in the rising cost of higher education and issues of access and affordability. It is relevant to the issues discussed at the recent White House Opportunity Summit on higher education.
American Higher Education and Income Inequality
February 2014

Introduction

One of the most important developments in the U.S. economy going forward that will affect higher education is the aggregate growth of real incomes. During the last decade, the real incomes of families across the income distribution suffered (Saez 2013.) This has been a major cause of the downward pressure on tuition increases and the increasing need for financial aid at many non-profit institutions of higher education.¹ And, in the public sector, slower income growth has affected tax revenues and commitments to public higher education. Constrained appropriations to public institutions have led to higher tuition, as schools attempt to protect quality. Going forward, as real income growth picks up, so will the ability of some institutions to increase tuition in both the private non-profit and the public sectors, as families would be able to afford higher expenditures on education. The need for financial aid would moderate, and the possibility of renewed public support through state budgets would also improve.²

In addition to the aggregate growth rate of income, the distribution of income across families also matters for colleges and universities. Income inequality has increased in the United States over the last several decades, with the share of income earned by the top 10% of the income

² There is some automatic adjustment to higher education institutions’ costs if real income growth slows. When real wages for skilled labor go down across the economy, putting downward pressure on tuition increases, this also puts downward pressure on compensation for skilled workers at colleges and universities. This adjustment can be painful, however, and slow given traditions of shared governance.
distribution increasing from about 33% of total income in the 1970s to a little over 48% in 2011.\textsuperscript{3} This has resulted in large part because the increase in demand for skilled labor resulting from skill biased technological change has exceeded the increase in supply of skilled labor (Goldin and Katz 2008.) Greater investment in human capital, through increased access to higher education, would help moderate the increase in income inequality. But, increasing income inequality itself is in fact exacerbating the challenges facing colleges and universities and making it more difficult for institutions of higher education to contribute to greater income equality.

Real income growth that is skewed toward higher income families creates challenges for higher education because the highest income families are willing and able to pay the full sticker price. And schools compete for these students, supplying the services that they desire, pushing up costs. At the same time, many schools have been committed to recruiting and educating a socioeconomically diverse student body, and thereby contributing to greater income equality over time.\textsuperscript{4} But, lower income families’ incomes have lagged even further behind the top groups, because of increased income inequality, increasing the need for financial aid. If the income distribution were less skewed, the demand for services at one end of the income distribution on the part of higher income families and the need for financial aid at the other end on the part of lower income families would both moderate, reducing the financial challenges facing many colleges and universities.

\textsuperscript{3} Saez, 2013. Much of this shift is explained by changes in the incomes of the top 1% of families.
\textsuperscript{4} At the private, non-profits this has occurred through financial aid policies, while at the publics, low tuition policies have historically supported access.
Over the last decade or so, there is been an increasing refrain from many quarters that the financial model of American higher education is broken and not sustainable. This mainly refers to the high tuition institutions that have been experiencing rising costs, rising tuition, and rising claims on financial aid. While this clearly applies to much of the non-profit sector, in particular the selective non-profits, it is also relevant to many institutions in the public sector as well, which have become increasingly less reliant on state appropriations by necessity. These phenomena are in part the result of the increasing income inequality in America.

**Summary**

This paper will show how increasing income inequality is contributing to the trends we see in American higher education and to the financial challenges that many colleges and universities are currently facing. It describes a simple model of how colleges and universities behave to demonstrate that increased income inequality, given the structure of many American higher education institutions, can contribute to these challenges. (A formal model is presented in the appendix.) The discussion is most appropriate for the selective non-profit higher education institutions, but is also relevant to the selective publics that do not admit all applicants.

A simple numerical example is presented that estimates how much lower tuition, spending (proxying for costs), and financial aid would have been, if household incomes in the United

---

5 As public institutions push up tuition, increase fund raising efforts, and offer greater financial aid, their financial model moves closer to that of the private, non-profits.

6 The response of open enrolment colleges and universities to increasing income inequality is likely to be different than the selective schools. Tuition at community colleges and less selective four-year publics would be determined more by the incomes of lower income families, who account for a larger share of their student demographic. If income inequality were decreased, and lower income families were doing better, the tuition levels at these school might be higher, as demand on the part of these families would be higher.
States had grown by the same aggregate amount between 1971 and 2009, but with no increase in income inequality.\textsuperscript{7} It is based on data from a set of selective non-profit colleges and universities, and most appropriate for demonstrating the impact of greater income equality on trends at selective colleges and universities over time.\textsuperscript{8}

The policy insight of this is that higher education is being held responsible by the press, politicians and the general public for trends that are in part the result of rising income inequality in America. And, the rising income inequality has many causes and implications, but the government is in the best position to respond to changes in income distribution, through a variety of policies at its disposal.

**How Colleges and Universities Make Decisions About Spending and Prices**

Much of the criticism targeted at American higher education is focused on the non-profit sector, particularly the selective schools with total student charges (the sticker price) above $50,000, as well as the higher cost, selective publics. While more students are enrolled in open admission two and four year public institutions of higher education, these selective schools nonetheless receive significant attention. This is in part because they are among the most selective and spend the most per student, not unrelated phenomena. A simple model can be designed to describe their decision-making in the market in which they operate. (See the appendix.) Based on Winston (2003), it is assumed that schools make decisions to increase the demand on the part of students and their families for places at their school. Unlike other markets, however, schools do not adjust

\textsuperscript{7} 2009 in used, rather than a more recent year, because of data availability.

\textsuperscript{8} The exercise could be extended to the selective public institutions as well. In this sector, increasing tuition levels have been importantly affected by constrained state appropriations as well as by rising income inequality, the primary focus of this paper.
price to equate supply with demand. In fact, schools attempt to create a queue of student applicants, from which they can admit the most qualified students. The admit rates at schools like Harvard, Yale, Princeton and Stanford are all below 10%, indicating the intensity of the excess demand. Selective publics also admit low shares of their applicant pools. This excess demand allows schools to control student quality through the admissions process. In a market where the consumers are also inputs into the educational process (Rothchild and White 1995), this contributes to the quality of the education offered. It is also assumed that schools, “part church” and “part car dealer” (Winston 2000), also care about socioeconomic diversity. This has increasingly been stated as an explicit objective of schools over the last forty years.⁹ So, schools make decisions in order to increase the demand for their school on the part of talented students, but also want to attract students from lower income families as an additional objective.

The decision making problem for a simplified, representative school is to maximize the demand on the part of both high income and low income student applicants, from which the school will decide who to admit, selecting the highest quality class from the two pools of applicants. The major decision variables under the school’s control are the amount it spends on programs and the prices it charges the high and low-income students. Tuition is the price that the high income students are asked to pay, and the net price is the price that the low income students are asked to pay and equals the tuition price that the high income students pay minus financial aid.¹⁰ The

---

⁹ See Bowen, Kurzweil, and Tobin, 2006, for a discussion of the commitment to socioeconomic diversity on the part of colleges and universities. This work, and others, including Hill, Van Atta, Gambhir, and Winston (2011) and Carnevale and Rose (2004) discuss the extent to which this stated objective has been accomplished.

¹⁰ The distinction between tuition and total student charges is ignored here. Tuition could more accurately be defined as total student charges. For simplicity, I am ignoring this distinction.
demand on the part of high income students depends on expenditures and tuition, while that of low income students depends on expenditures and the net price. The school is subject to a budget constraint, which depends on its decisions about prices and expenditures, the number of high income and low income students it matriculates, as well as its other revenue sources, including its assets or endowment.

The demand on the part of students and their families also depends on their incomes. The higher their incomes, the greater the demand (willingness and ability to pay) for education for their children.\(^{11}\) The school faces given income levels for its applicants (both high and low income families) and given endowment levels and makes decisions about the desired socioeconomic diversity of its student body. It then decides on expenditures, tuition, and net price to maximize the demand on the part of high and low income students, to generate the queues from which the school will pick the most qualified students. This is a highly simplified description of how the selective, non-profit schools behave, but is useful for exploring some of the important decisions that schools must make. (For selective publics, state appropriations could be added to the budget constraint, and endowment returns would play a less significant role.) Given the incomes of their applicant pools and the school’s budget constraint, they decide how much to spend, what to charge full pay students and how much financial aid to offer, to maximize the quality of their student bodies subject to their objectives for socioeconomic diversity.\(^{12}\)

\(^{11}\) The model assumes that for a large segment of higher education, to which the model applies, there is excess demand for access to these schools, suggesting that there is a positive return to receiving an education from these institutions.

\(^{12}\) One could model this in different ways. Schools could face a continuous distribution of students by both talent and income and seek to maximize the quality of their students at the least cost. The model as presented is designed to both simplify the presentation, and to demonstrate the pressures that schools will face when they care about attracting both a
Schools will allocate their resources on the margin to increase the demand on the part of the two types of students they desire to recruit. Those students care about how much the school spends on their education, and it is assumed that schools cannot target specific expenditures to particular types of students. (This could be relaxed and overstates the extent to which expenditures are available equally to all students. Some expenditures probably benefit, or are used by, different types of students. Examples might include expenditures on the sailing club versus a variety of support services that might be targeted at first-generation, lower income students.) They also care about the price they are asked to pay. In equilibrium, the school will have made decisions about how much to spend and how much to charge each group such that an additional dollar of expenditure generates the same amount of extra demand on the part of the two groups of students as would result if that dollar were used instead to reduce the prices faced by both groups, and the price reductions in tuition and net price would be such that, on the margin, they generate equal changes in demand on the part of the two types of students.

These decisions about how much to spend and what to charge different students are among the most important decisions that schools make. Schools decide how much to spend and on what, to supply as high a quality education as possible, captured in the model as generating a queue of talented students who want access to that education. But, students and families also care about talented and a socioeconomically diverse student body. The model as presented assumes there is talent in both pools of students, high income and low income, but that the school is not indifferent between these students. It desires students from both groups, so as to meet its objectives of contributing to social mobility in the U.S. and to learning by bringing together students with different life experiences.

Whether the views of families and students on a quality education and that of the school are the same is ignored for now. The model could certainly help explain why schools spend on amenities that they perhaps do not believe add fundamentally to the quality of the
the price they have to pay. Schools, on the margin, are deciding whether an additional dollar would be better spent on program or on reducing price for the students it is trying to attract. Prices are higher for high income students, because they are willing and able to pay more and would rather do so than attend a school that spent less on their education. (Such a school is in fact in their choice set, in the form of less selective alternatives, with less spending per student and a lower price.) Lower income families are less willing or able to pay for education for their children, so on the margin face a different trade-off between spending and price. The level of expenditure and prices set by schools reflect this.  

Implications for Behavior in Response to Changes in Assets or Income Level  

Colleges and universities make decisions, taking their wealth and the incomes of their applicant pools as given at any one time. If their wealth or the incomes of the students applying change, their decisions about spending and pricing also change. For example, an exogenous increase in a school’s assets relaxes the budget constraint facing the school. This could result from a gift or unexpected appreciation of the endowment.

Relaxing the budget constraint increases spending and puts downward pressure on tuition and net prices. The results on price may not seem intuitive to any observer of higher education, when price increases have been persistent year in and year out. But, all else constant, greater resources

---

14 This model could easily be extended to incorporate merit aid.
15 While the model has been developed with the private non-profit colleges and universities in mind, it also could be used for selective publics. In this case, the budget constraint would include state appropriations, which would work exactly like changes in earnings on the endowment in the model.
on the part of schools lead them to make decisions on the margin between increasing spending and reducing price (or increasing it by less, given other changes it faces) so as to increase demand on the part of applicants. Increased spending on financial aid, reducing net price, is more intuitive, in response to increased wealth and is certainly consistent with observed behavior on the part of many colleges and universities in the 2000s, before the financial market crisis of 2008/09.\textsuperscript{16}

It is also possible to look at the effects of a change in income on the part of high income or low income families. When high income families’ incomes increase, their demand for education increases. Schools see an increase in demand for their services on the part of these families. This pushes their decisions about spending and pricing out of equilibrium, with greater than expected demand on the part of high income families on the margin. They respond to the increase in demand by increasing the price charged the higher income students, tuition, which relaxes the school’s budget constraint, allowing an increase in expenditure. This increase in expenditure increases the demand on the part of low-income students, allowing an increase in net price as well. But, tuition increases by more than net price, resulting in an increase in financial aid. These changes restore equilibrium. Summarizing, an increase in income on the part of high income families leads to an increase in tuition, in spending (costs) and financial aid.

\textsuperscript{16} Because the effects of changes in state appropriations for selective publics works exactly like a change in assets in the model as specified, a cut in state appropriations would put downward pressure on expenditures and upward pressure on tuition and net prices, consistent with the experiences at many public institutions.
The intuition for a change in low-income family income is of course similar. Greater incomes on the part of low income families would lead to increased spending, increases in net price greater than tuition, and therefore moderation in financial aid.\textsuperscript{17}

\textbf{The Impact of Increasing Income Inequality in the United States on Higher Education}

It is possible to examine the impact on the equilibrium levels of expenditures, prices and net prices of a change in income distribution between the high and low income families. This is demonstrated by looking at the effects on expenditure, tuition and net price of a change in high income and low income family incomes, such that the increase in high incomes is offset by a reduction in low incomes. This represents an unchanged aggregate income, but increased income inequality.

It is possible to show that expenditures would go up, tuition would go up, and financial aid would go up, as a result of the increase in tuition and the fall in incomes of the lower income families. (See appendix.) This result depends on there being differences between high income and low income families in their demand for education. If they were exactly the same, of course, a change in the distribution of income would have no impact. An assumption sufficient to guarantee this result is that lower income families are more price sensitive than higher income families. This is consistent with the existing empirical evidence.\textsuperscript{18} In this case, as higher income families’ incomes go up, they demand more education, while lower income families’ incomes

\textsuperscript{17} The effects of a recession on colleges and universities could also be explained, by looking at the effects of declines in incomes for all families, plus any impacts on colleges’ and universities’ budget constraints through effects on endowments, gifts, or state appropriations.

\textsuperscript{18} See McPherson, Schapiro and Winston (1993).
decline, reducing their demand. These effects lead to a larger increase in tuition for higher income families than reduction in net price for low income families to restore equilibrium, given the different price sensitivities. The new equilibrium, with the same total income but greater income inequality, will involve higher tuition, financial aid, and spending (because the tuition increase is greater than the fall in net price, leading to an increase in net revenue and a relaxation of the budget constraint). 19

Numerical Illustration

A simple numerical example can also demonstrate the impact of rising income inequality in the United States on the private, non-profit segment of the higher education sector. Using household incomes in 1971 and 2009, it is possible to estimate what household incomes would have been if household incomes had grown at the same rate across the income distribution as in the aggregate. This would have left the income distribution unchanged at its 1971 distribution, rather than growing more unequal with income growing more rapidly among higher income households.

Table 1 shows the income bounds by income quintile and for the top 5% of the income distribution for household income for 1971 and 2009 (in current dollars). The ratio of these two numbers across the income distribution for 2009 is one way of seeing the increased income

19 The results require that the income elasticity of demand for high and low-income students be positive. Significant differences in income elasticity of demand between high and low-income families wouldn't change the results for tuition or net price, but could affect the results for expenditures, through the budget constraint. There is not much empirical evidence on differences in the income elasticity for demand for higher education by income level. If one thinks of the education offered by the selective schools as a luxury good for higher income families, their income elasticity could be higher, making the result for expenditures as discussed in the text more likely. Again, the results would hold for tuition and net price even if this were not the case.
inequality that has taken place in the United States. The lower bound of the top 20% of the income distribution and that of the top 5% have gone up relative to the other income quintile boundaries, by 6.6 and 7.5 fold compared to 5.4, 5.3 and 5.8 for the top boundaries of the first, second and third quintiles respectively. Table 1 also estimates what those bounds would have been in 2009 had the distribution of income remained unchanged since 1971, assuming aggregate income still grew by the same amount.\(^{20}\)

Hill, Van Atta, Gambhir, and Winston (2011) reports data for a set of 30 highly selective colleges and universities for 2008/09, including sticker prices, net prices by income quintile, as well as the shares of students in these schools by income quintile. We can use these data to explore the impact of increasing income inequality on the selective non-profit institutions. First, as discussed above, if the incomes of higher income families had increased by less, it is quite likely that tuition increases would have been more moderate. Hill et. al. find that 69% of students at this group of schools come from the top 20% of the income distribution, and that 51% pay the full sticker price. There are two simple ways to estimate what might have happened to tuition growth had income growth among the top income families been more moderate. First, assuming that the sticker price would have remained at the same ratio of the income level of families at the top 5% of the income distribution, the sticker price in 2009 would have been $30,788 rather than $35,070. Alternatively, we could assume that these high income families would have been willing to allocate the same share of the (smaller) increase in their incomes to tuition increases as they did with the actual growth of their incomes. This leads to a very similar number of $30,537. (See Archibald and Feldman (2011) who use this as a definition of affordability.) It is also

\(^{20}\) If everyone’s income grew at the aggregate rate, then the boundaries would have grown at this rate and income distribution would have remained unchanged.
possible to estimate what schools would have spent on financial aid and how much they would have had available through net revenues to spend on their educational programs under a more equal income distribution. Again, using the data from Hill et. al., assume that the share of students from each income quintile remained unchanged at the actual levels in 2009. (See Table 2.) We can either assume that the net price that they were asked to pay remained the same relative to the sticker price paid by the full pay students, or the same relative to their quintile’s median income. Financial aid would have fallen respectively by 12 or 22% under these two assumptions. Financial aid falls by more under the assumption that the net price depends on quintile median income because with more equal income distribution assumed, quintile median incomes are higher, resulting in a higher net price than in the scenario assuming that the net price remains at the same ratio of the original sticker price. We can also calculate the impact on net revenues. Tuition revenues from full pay students fall as a result of the smaller increase in tuition growth rates as a result of less income inequality, but financial aid also declines, because of both the slower increase in tuition and the higher incomes assumed on the part of lower income families. Taking these together, however, net revenues decline, leaving schools with fewer resources to push up expenditures and cost. Under the two alternative assumptions about what happens to net prices for financial aid students, net revenues would have fallen by 13% and 9% respectively.

This simple example suggests that many of the troubling trends facing this sector of American higher education result in part from the increasing income inequality in America.\textsuperscript{21} It is not just

\textsuperscript{21} The empirical exercise is not meant to explain definitively the size of cost, tuition and net price changes across higher education. It is meant to demonstrate that the model and some
the growth of aggregate income that matters to these schools, but its distribution across families. This results from these schools’ commitment to recruiting a socioeconomically diverse student body. As income inequality increases, tuition and spending go up in response to rising incomes for higher income families, while financial aid rises to continue to attract lower income students.

Because these trends are shown to be the outcome of a school’s optimizing decisions, they are by definition sustainable. But, this takes as given the school’s commitment to socioeconomic diversity, assuming the school is committed to attracting given numbers of both high and low income students. As income inequality increases, the cost of attracting a low income student rises relative to the high income student. As a result, schools’ commitment to socioeconomic diversity will be tested by increasing income inequality. Increasing income inequality also contributes to increasing skill gaps between high income and low income students over time (see Altonji, Bharadwaj, and Lange 2012), making it more difficult to attract talented low income students into the applicant pool. This means that lower income students will increasingly involve a cost not only in terms of the price their families can pay, but in terms of the attributes or level of skills they bring to the student body.

**Conclusion**

The increasing income inequality in the United States can reasonably be demonstrated to have contributed to increasing tuition, increasing spending, and greater financial aid at many colleges and universities that attract both a talented and socioeconomically diverse student body. These developments have been cited as reasons to believe that the financial model facing these simple assumptions, given the data available, suggest that the impact of increasing income inequality is not trivial.
institutions is not sustainable. But, rather than being a criticism of these institutions, it is in part a result of their response to rising income inequality in the United States, as they continue to try to attract students from across the income distribution. These schools have been criticized for their high tuitions and high spending, but this is the result of the demand for services on the part of high income families, who have done exceptionally well over the last several decades, with increasing income inequality. To ask schools to restrain tuition and spending in the face of this demand is difficult. Students will go to the schools that meet their demands. They are willing and able to pay for the expensive education offered by these schools.

The government is in the best position to address rising income inequality directly, through the tax system and by increasing investment in education, which would alleviate some of the pressures facing colleges and universities. At the same time, policy makers could change the incentives that schools face in setting tuition, financial aid and expenditures, to help insure that the education system contributes to, rather than worsens, future income inequality. This needs to be done carefully to create the desired incentives and avoid unintended consequences. For example, there is currently pressure on schools to moderate tuition increases, with some confusion about the impact of this on access issues. If schools compensate for lower tuition revenues by cutting financial aid rather than recuing costs, socioeconomic diversity will suffer, not improve.  

22 We are seeing this in the private, non-profit sector. Lower tuition combined with lower financial aid benefits higher income students, contributing to worsening income inequality in the future.
Absent changes in policies, existing research suggests we will see continued increases in income inequality (Altonji, et. al. 2012.) This will continue to create challenges for colleges and universities that are committed to attracting talented students from across the income distribution and contributing to social mobility and equal opportunity.
REFERENCES


Table 1: Increase in Income inequality in America

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Top 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper boundary (current dollars)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>$3,800</td>
<td>$7,244</td>
<td>$10,660</td>
<td>$15,200</td>
<td>na</td>
<td>$24,138</td>
</tr>
<tr>
<td>2009</td>
<td>$20,453</td>
<td>$38,550</td>
<td>$61,801</td>
<td>$100,000</td>
<td>na</td>
<td>$180,001</td>
</tr>
<tr>
<td>2009/1971</td>
<td>5.4</td>
<td>5.3</td>
<td>5.8</td>
<td>6.6</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Proj. 2009</td>
<td>$24,878</td>
<td>$47,425</td>
<td>$69,789</td>
<td>$99,512</td>
<td>na</td>
<td>$158,027</td>
</tr>
</tbody>
</table>

Notes: The projected income quintile boundaries are estimated by assuming that they grew at the rate of average household income between 1971 and 2009.

Table 2: Shares of students and net price by quintile: 30 Selective Private Colleges and universities (2008/09)

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Top 5% (Full pay)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of students</td>
<td>4.6</td>
<td>6.8</td>
<td>7.8</td>
<td>11.6</td>
<td>18.1</td>
<td>51.2</td>
</tr>
<tr>
<td>Net price as % of sticker price</td>
<td>6</td>
<td>8</td>
<td>18</td>
<td>34</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>Net prices as % of Quintile median income</td>
<td>17</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>17</td>
<td>24</td>
</tr>
</tbody>
</table>

Note: These data are used to estimate what would have happened to tuition, net prices, and net revenues had Income inequality not increased between 1971 and 2009.
Appendix: The Model

The Optimization Problem

The objective function of a simplified, representative school is to maximize the demand on the part of both high income and low income student applicants, from which the school will decide who to admit. For simplicity, assume there are just two types of students, high-income and low-income, whose demands are represented by $D^H$ and $D^L$. For simplicity, it is also assumed that the school has decided on the quantity of each type of student, high and low income, that it wants in the class, $Q^H$ and $Q^L$. It then makes decisions to increase the demand on the part of both groups, so that it can select the highest quality class from the two pools of applicants. It is assumed that schools spend a fixed percent each year, $r$, from their assets, $A$. A more complicated spending rule, and current gifts and other sources of current revenue could easily be included in the model.

The constrained optimization problem facing a representative school becomes:

Maximize $D^H(E,T,Y^H) + D^L(E,NP,Y^L)$ subject to $T*Q^H + NP*Q^L + rA - E = 0$

Or,

Max $L = D^H(E,T,Y^H) + D^L(E,NP,Y^L) + \lambda(T*Q^H + NP*Q^L + rA - E)$

Where $D^H, D^L$ is the demand on the part of high (low) income students

$E$ is the school’s expenditure on program, which can be equally accessed by both high income and low income students.

$T$ is the full sticker price, paid by the high income students

$NP$ is the net price paid by the lower income students

$Q^H, Q^L$ are the fixed numbers of high and low income students that the school desires for socioeconomic diversity

$Y^H, Y^L$ are the incomes of the high and low income families

$rA$ refers to the earnings on the school’s endowment, which contributes to the amount that the school can spend each year.
First Order Conditions

Differentiating the Lagrangian with respect to E, T, NP and λ and setting the results equal to 0 yields the following:

\[
\frac{\partial L}{\partial \lambda} = rA + TQ^H + NPQ^L - E = 0
\]

\[
\frac{\partial L}{\partial T} = \frac{\partial D^H}{\partial T} + \lambda Q^H = 0
\]

\[
\frac{\partial L}{\partial NP} = \frac{\partial D^L}{\partial NP} + \lambda Q^L = 0
\]

\[
\frac{\partial L}{\partial E} = \frac{\partial D^H}{\partial E} + \frac{\partial D^L}{\partial E} - \lambda = 0
\]

or,

\[
\frac{\partial D^H}{\partial E} + \frac{\partial D^L}{\partial E} = -\frac{\partial D^H}{\partial T} = -\frac{\partial D^L}{\partial NP}
\]

assuming $Q^H = Q^L = 1$ for simplicity.

SECOND ORDER CONDITIONS:

Totally differentiating the four first order conditions yields:

\[
rdA + Q^HdT + Q^LdNP - dE = 0
\]

\[
\frac{\partial^2 D^H}{\partial T \partial E} dE + \frac{\partial^2 D^H}{\partial T^2} dT + \frac{\partial^2 D^H}{\partial T \partial Y^H} dY^H + Q^Hd\lambda = 0
\]

\[
\frac{\partial^2 D^L}{\partial NP \partial E} dE + \frac{\partial^2 D^L}{\partial NP^2} dNP + \frac{\partial^2 D^L}{\partial NP \partial Y^L} dY^L + Q^Ld\lambda = 0
\]
\[
\frac{\partial^2 D^H}{\partial E^2} \, dE + \frac{\partial^2 D^H}{\partial E \partial T} \, dT + \frac{\partial^2 D^H}{\partial E \partial Y^H} \, dY^H - d\lambda + \frac{\partial^2 D^L}{\partial E^2} \, dE + \frac{\partial^2 D^L}{\partial E \partial NP} \, dNP + \frac{\partial^2 D^L}{\partial NP \partial Y^L} \, dY^L = 0
\]

Rearranging yields the following:

\[
\begin{bmatrix}
0 & Q^H & Q^L & -1 \\
\frac{\partial^2 D^H}{\partial T^2} & 0 & \frac{\partial^2 D^H}{\partial T \partial E} & \frac{\partial^2 D^H}{\partial T \partial Y^H} \\
0 & \frac{\partial^2 D^L}{\partial NP^2} & \frac{\partial^2 D^L}{\partial NP \partial E} & \frac{\partial^2 D^L}{\partial NP \partial Y^L} \\
-1 & \frac{\partial^2 D^H}{\partial E \partial T} & \frac{\partial^2 D^L}{\partial E \partial NP} & \frac{\partial^2 D^H}{\partial E^2} + \frac{\partial^2 D^L}{\partial E^2}
\end{bmatrix}
\begin{bmatrix}
d\lambda \\
dT \\
dNP \\
dE
\end{bmatrix}
= 0
\]

\[
\begin{bmatrix}
0 & 0 & -r \\
-\frac{\partial^2 D^H}{\partial T \partial Y^H} & 0 & 0 & \frac{dY^H}{dE^2} \\
0 & -\frac{\partial^2 D^L}{\partial NP \partial Y^L} & 0 & \frac{dY^L}{dE^2} \\
-\frac{\partial^2 D^H}{\partial E \partial Y^H} & -\frac{\partial^2 D^L}{\partial E \partial Y^L} & 0 & \frac{dA}{dE^2}
\end{bmatrix}
\begin{bmatrix}
dT \\
dNP \\
dE \\
dA
\end{bmatrix}
= 0
\]

The second order conditions for the constrained optimization are satisfied if the \(d^2z\) is negative definite subject to \(dg = 0\), where \(z\) is the function to be maximized and \(g\) is the constraint. Using the 4x4 matrix on the left hand side, this will be the case if \(|H_2|>0\) and \(|H_3|<0\), where the former is the 3x3 bordered hessian matrix and the latter is the 4x4 bordered hessian matrix. (See Chiang, 1984, pp. 384-5).
Sufficient conditions for the second order conditions to hold for a constrained maximum are:

\[ \frac{\partial^2 D^H}{\partial E^2} + \frac{\partial^2 D^L}{\partial E^2} < 0 \]

\[ \frac{\partial^2 D^L}{\partial NP^2}, \frac{\partial^2 D^H}{\partial T^2} < 0 \]

(This means that increases in price, for both high and low income students, reduce demand by greater absolute amounts as prices increase. In other words, the negative number increases in absolute value as price increases. This is necessary for the 3x3 matrix to be positive, needed for the second order conditions to be satisfied.)

and

\[ \frac{\partial^2 D^L}{\partial E \partial NP} \cdot \frac{\partial^2 D^H}{\partial E \partial T} = 0 \]

(This condition is not necessary, but makes it possible to unambiguously sign the 4x4 matrix, and is sufficient to make \( d^2z \) subject to \( dg=0 \) be negative definite.)

**COMPARATIVE STATIC RESULTS**

The comparative static results can be derived using Cramer’s Rule on the above.

Assuming that the second order conditions for a maximum are met and that some of the second derivatives equal zero or of a particular relative magnitude, it is possible to demonstrate that:

\[ \frac{dE}{dA} > 0 \text{ while } \frac{dT}{dA}, \frac{dNP}{dA} < 0. \]

Totally differentiating the first order conditions with respect to \( Y^H \) and \( Y^L \) and rearranging yields:
\[
\frac{dE}{dY^H} > 0 \quad \frac{dT}{dY^H} > 0 \quad \text{and} \quad \frac{dNP}{dY^H} < \frac{dT}{dY^H}
\]

and

\[
\frac{dE}{dY^L} > 0 \quad \frac{dNP}{dY^L} > 0 \quad \text{and} \quad \frac{dNP}{dY^L} > \frac{dT}{dY^L}
\]

In addition to the above assumptions for the second order condition to hold, the following assumptions are made in deriving the results:

\[
\frac{\partial^2 D^H}{\partial E \partial Y^H}, \quad \frac{\partial^2 D^L}{\partial E \partial Y^L} > 0
\]

\[
\frac{\partial^2 D^H}{\partial T \partial Y^H}, \quad \frac{\partial^2 D^L}{\partial NP \partial Y^L} > 0
\]

**Increased Income Inequality**

The effect of increased income inequality on \( E, T \) and \( NP \) is derived by looking at the combined effects of:

\[ dY^H = -dY^L \]

In deriving the effects of increased income inequality on the equilibrium, I assumed the following:

All second derivatives the same for high income and low income families, except:

\[
\frac{\partial^2 D^L}{\partial NP^2} < \frac{\partial^2 D^H}{\partial T^2}, \quad \text{or that the effect of an increase in net price for low income families on demand is a bigger absolute value than for high income families.}
\]

If the following inequalities hold, the results would remain unchanged:
\[ \frac{\partial^2 D^H}{\partial E \partial Y^H} > \frac{\partial^2 D^L}{\partial E \partial Y^L} \]

\[ \frac{\partial^2 D^H}{\partial T \partial Y^H} > \frac{\partial^2 D^L}{\partial N \partial Y^L} \]

**Extensions to the Model**

Several simplifying assumptions have been made that could be relaxed. It is assumed that schools have made commitments to attract fixed numbers both high income and low income students. The share of high income and low income students could instead be determined by the model, given the characteristics of these students. The model, as formulated, assumes that the school has already done this independently, and decided on the ratio of students in these two groups given their desirability (based on their talents and their socioeconomic diversity, given the school’s objectives), and then makes decisions to increase the queues of both students from which it chooses the fixed number from each group.

It would also be possible to allow expenditures to be targeted toward each of the type of students. In general, however, most expenditures are not or cannot be targeted in this way. The logic of the model would suggest that, if this were relaxed, schools would allocate expenditures to generate the most demand on the part of the two groups, equating on the margin the impact on demand of expenditures targeted to each group.

The budget constraint could also be made more complicated. As written, the intertemporal decisions that face institutions has been simplified by assuming that schools spend a fixed share of their endowments each year, modeled as a very simplistic spending rule. A more complicated
two period version of the model could be used to examine the decisions that schools have to make about spending resources today versus saving them for the future. A more complicated objective function that included the impact of decisions today on the demand of students tomorrow, for example through reputation effects or rankings, would allow an extension of the model to be used to address a different set of questions.

Finally, each school is assumed to take the behavior of other schools as given. Changes in other schools’ behavior could be included as exogenous variables in the model as presented. Essentially, schools are assumed to be small in the higher education market place.